2023

AP® Calculus AB

Scoring Guidelines

Part A (AB or BC): Graphing calculator required

Question 1 — 9 points

General Scoring Notes

The model solution is presented using standard mathematical notation.

Answers (numeric or algebraic) need not be simplified. Answers given as a decimal approximation should be correct to three places after the decimal point. Within each individual free-response question, at most one point is not earned for inappropriate rounding.

t (seconds)	0	60	90	120	135	150
$f(t)$ (gallons per second)	0	0.1	0.15	0.1	0.05	0

A customer at a gas station is pumping gasoline into a gas tank. The rate of flow of gasoline is modeled by a differentiable function f, where $f(t)$ is measured in gallons per second and t is measured in seconds since pumping began. Selected values of $f(t)$ are given in the table.

	Model Solution	Scoring	
(a)	Using correct units, interpret the meaning of $\int_{60}^{135} f(t)\,dt$ in the context of the problem. Use a right Riemann sum with the three subintervals $[60, 90]$, $[90, 120]$, and $[120, 135]$ to approximate the value of $\int_{60}^{135} f(t)\,dt$.		
	$\int_{60}^{135} f(t)\,dt$ represents the total number of gallons of gasoline pumped into the gas tank from time $t = 60$ seconds to time $t = 135$ seconds.	Interpretation with units	**1 point**
	$\int_{60}^{135} f(t)\,dt$ $\approx f(90)(90-60) + f(120)(120-90) + f(135)(135-120)$ $= (0.15)(30) + (0.1)(30) + (0.05)(15) = 8.25$	Form of Riemann sum	**1 point**
		Answer	**1 point**

Scoring notes:

- To earn the first point the response must reference gallons of gasoline added/pumped and the time interval $t = 60$ to $t = 135$.
- To earn the second point at least five of the six factors in the Riemann sum must be correct.
- If there is any error in the Riemann sum, the response does not earn the third point.
- A response of $(0.15)(30) + (0.1)(30) + (0.05)(15)$ earns both the second and third points, unless there is a subsequent error in simplification, in which case the response would earn only the second point.

- A response that presents a correct value with accompanying work that shows the three products in the Riemann sum but does not show all six of the factors and/or the sum process, does not earn the second point but does earn the third point. For example, responses of either $4.5 + 3.0 + 0.75$ or $(0.15)(30), 0.1(30), 0.05(15) \rightarrow 8.25$ earn the third point but not the second.
- A response of $f(90)(90-60) + f(120)(120-90) + f(135)(135-120) = 8.25$ earns both the second and the third points.
- A response that presents an answer of only 8.25 does not earn either the second or third point.
- A response that provides a completely correct left Riemann sum with accompanying work, $f(60)(30) + f(90)(30) + f(120)(15) = 9$, or $(0.1)(30) + (0.15)(30) + (0.1)(15)$ earns 1 of the last 2 points. A response with any errors or missing factors in a left Riemann sum earns neither of the last 2 points.

Total for part (a) 3 points

(b) Must there exist a value of c, for $60 < c < 120$, such that $f'(c) = 0$? Justify your answer.

f is differentiable. $\Rightarrow$ f is continuous on $[60, 120]$.	$f(120) - f(60) = 0$	**1 point**
$\dfrac{f(120) - f(60)}{120 - 60} = \dfrac{0.1 - 0.1}{60} = 0$ By the Mean Value Theorem, there must exist a c, for $60 < c < 120$, such that $f'(c) = 0$.	Answer with justification	**1 point**

Scoring notes:

- To earn the first point a response must present either $f(120) - f(60) = 0$, $0.1 - 0.1 = 0$ (perhaps as the numerator of a quotient), or $f(60) = f(120)$.
- To earn the second point a response must:
  - o have earned the first point,
  - o state that f is continuous because f is differentiable (or equivalent), and
  - o answer "yes" in some way.
- A response may reference either the Mean Value Theorem or Rolle's Theorem.
- A response that references the Intermediate Value Theorem cannot earn the second point.

Total for part (b) 2 points

(c) The rate of flow of gasoline, in gallons per second, can also be modeled by $g(t) = \left(\frac{t}{500}\right)\cos\left(\left(\frac{t}{120}\right)^2\right)$ for $0 \le t \le 150$. Using this model, find the average rate of flow of gasoline over the time interval $0 \le t \le 150$. Show the setup for your calculations.

$\dfrac{1}{150 - 0}\displaystyle\int_0^{150} g(t)\,dt$	Average value formula	**1 point**
$= 0.0959967$	Answer	**1 point**

The average rate of flow of gasoline, in gallons per second, is 0.096 (or 0.095).	

Scoring notes:

- The exact value of $\frac{1}{150}\int_0^{150} g(t)\,dt$ is $\frac{12}{125}\sin\left(\frac{25}{16}\right)$.
- A response may present the average value formula in single or multiple steps. For example, the following response earns both points: $\int_0^{150} g(t)\,dt = 14.399504$ so the average rate is 0.0959967.
- A response that presents the average value formula in multiple steps but provides incorrect or incomplete communication (e.g., $\int_0^{150} g(t)\,dt = \frac{14.399504}{150} = 0.0959967$) earns 1 out of 2 points.
- A response of $\int_0^{150} g(t)\,dt = 0.0959967$ does not earn either point.
- Degree mode: A response that presents answers obtained by using a calculator in degree mode does not earn the first point it would have otherwise earned. The response is generally eligible for all subsequent points (unless no answer is possible in degree mode or the question is made simpler by using degree mode). In degree mode, $\frac{1}{150}\int_0^{150} g(t)\,dt = 0.149981$ or 0.002618.

Total for part (c) 2 points

(d) Using the model g defined in part (c), find the value of $g'(140)$. Interpret the meaning of your answer in the context of the problem.

$g'(140) \approx -0.004908$ $g'(140) = -0.005$ (or -0.004)	$g'(140)$	**1 point**
The rate at which gasoline is flowing into the tank is decreasing at a rate of 0.005 (or 0.004) gallon per second per second at time $t = 140$ seconds.	Interpretation	**1 point**

Scoring notes:

- The exact value of $g'(140)$ is $\frac{1}{500}\cos\left(\frac{49}{36}\right) - \frac{49}{9000}\sin\left(\frac{49}{36}\right)$.
- The value of $g'(140)$ may appear only in the interpretation.
- To be eligible for the second point a response must present some numerical value for $g'(140)$.
- To earn the second point the interpretation must include “the rate of flow of gasoline is changing at a rate of [the declared value of $g'(140)$]” and “at $t = 140$” (or equivalent).
- An interpretation of “decreasing at a rate of -0.005” or “increasing at a rate of 0.005” does not earn the second point.
- Degree mode: In degree mode, $g'(140) = 0.001997$ or 0.00187.

Total for part (d) 2 points

Total for question 1 9 points

Part A (AB): Graphing calculator required
Question 2 — 9 points

General Scoring Notes
The model solution is presented using standard mathematical notation.

Answers (numeric or algebraic) need not be simplified. Answers given as a decimal approximation should be correct to three places after the decimal point. Within each individual free-response question, at most one point is not earned for inappropriate rounding.

Stephen swims back and forth along a straight path in a 50-meter-long pool for 90 seconds. Stephen's velocity is modeled by $v(t) = 2.38e^{-0.02t}\sin\left(\frac{\pi}{56}t\right)$, where t is measured in seconds and $v(t)$ is measured in meters per second.

	Model Solution	Scoring	
(a)	Find all times t in the interval $0 < t < 90$ at which Stephen changes direction. Give a reason for your answer.		
	For $0 < t < 90$, $v(t) = 0 \Rightarrow t = 56$.	Considers sign of $v(t)$	**1 point**
	Stephen changes direction when his velocity changes sign. This occurs at $t = 56$ seconds.	Answer with reason	**1 point**

Scoring notes:

- A response that considers $v(t) = 0$ earns the first point.
- A response of "Stephen changes direction when his velocity changes sign" earns the first point for considering the sign of $v(t)$ but must include the answer of $t = 56$ in order to earn the second point.
- A response of $t = 56$ with no supporting work does not earn either point.
- Any presented values of t outside the interval $0 < t < 90$ will not affect scoring.

Total for part (a) — 2 points

	Model Solution	Scoring	
(b)	Find Stephen's acceleration at time $t = 60$ seconds. Show the setup for your calculations, and indicate units of measure. Is Stephen speeding up or slowing down at time $t = 60$ seconds? Give a reason for your answer.		
	$v'(60) = a(60) = -0.0360162$	$a(60)$ with setup	**1 point**
	Stephen's acceleration at time $t = 60$ seconds is -0.036 meter per second per second.	Acceleration units	**1 point**

$v(60) = -0.1595124 < 0$ Stephen is speeding up at time $t = 60$ seconds because Stephen's velocity and acceleration are both negative at that time.	Speeding up with reason	**1 point**

Scoring notes:

- The minimum work needed to earn the first point is $v'(60) = -0.036.$
  - $a(60) = -0.0360162$ is not sufficient to earn the first point. The connection $v'(t) = a(t)$ or $v'(60) = a(60)$ must be explicitly shown.
- A response must declare a value for $a(60)$ to be eligible for the second point.
- In order to earn the third point the presented conclusion must be consistent with a negative velocity at time $t = 60$ and the presented value of $a(60)$.
- A response does not need to find the value of $v(60);$ an implied sign is sufficient.
  - The statement "Stephen is speeding up because $a(60)$ and $v(60)$ have the same sign" (or equivalent) earns the third point, provided a negative value is presented for $a(60)$.
- A response that reports an incorrect sign or value of $v(60)$ does not earn the third point. Any presented value of $v(60)$ must be correct for the number of digits presented, from one up to three decimal places in order to earn the third point.
- Degree mode: A response that presents answers obtained by using a calculator in degree mode does not earn the first point it would have otherwise earned. The response is eligible for all subsequent points (unless no answer is possible in degree mode or the question is made simpler by using degree mode).
  - In degree mode, there are two possible values for $v'(60)$. A response that declares $v'(60) = -0.000141$ does not earn the first point but would earn the third point in the presence of $v(60) = 0.042089$ or $v(60) > 0$ and the conclusion that Stephen is slowing down.
  - Similarly, a response that declares $v'(60) = 0.039304$ does not earn the first point but would earn the third point in the presence of $v(60) = 0.042089$ or $v(60) > 0$ and the conclusion that Stephen is speeding up.

Total for part (b) 3 points

(c) Find the distance between Stephen's position at time $t = 20$ seconds and his position at time $t = 80$ seconds. Show the setup for your calculations.

$\int_{20}^{80} v(t)\,dt$	Integral	**1 point**
$= 23.383997$ The distance between Stephen's positions at $t = 20$ seconds and $t = 80$ seconds is 23.384 (or 23.383) meters.	Answer	**1 point**

Scoring notes:

- The first point is earned only for $\int_{20}^{80} v(t)\,dt$ (or the equivalent) with or without the differential.
- The second point is earned only for an answer of 23.384 (or 23.383) regardless of whether the first point was earned.
- Degree mode: In degree mode, $\int_{20}^{80} v(t)\,dt = 2.407982.$

Total for part (c) 2 points

(d) Find the total distance Stephen swims over the time interval $0 \le t \le 90$ seconds. Show the setup for your calculations.

$\int_{0}^{90} \lvert v(t) \rvert\,dt$	Integral	**1 point**
$= 62.164216$ The total distance Stephen swims over the time interval $0 \le t \le 90$ seconds is 62.164 meters.	Answer	**1 point**

Scoring notes:

- The first point is earned only for $\int_{0}^{90} \lvert v(t) \rvert\,dt$ or $\int_{0}^{56} v(t)\,dt - \int_{56}^{90} v(t)\,dt$ (or the equivalent), with or without the differential(s).
- The second point is earned only for an answer of 62.164 regardless of whether the first point was earned.
- Degree mode: In degree mode, $\int_{0}^{90} \lvert v(t) \rvert\,dt = 3.127892.$

Total for part (d) 2 points

Total for question 2 9 points

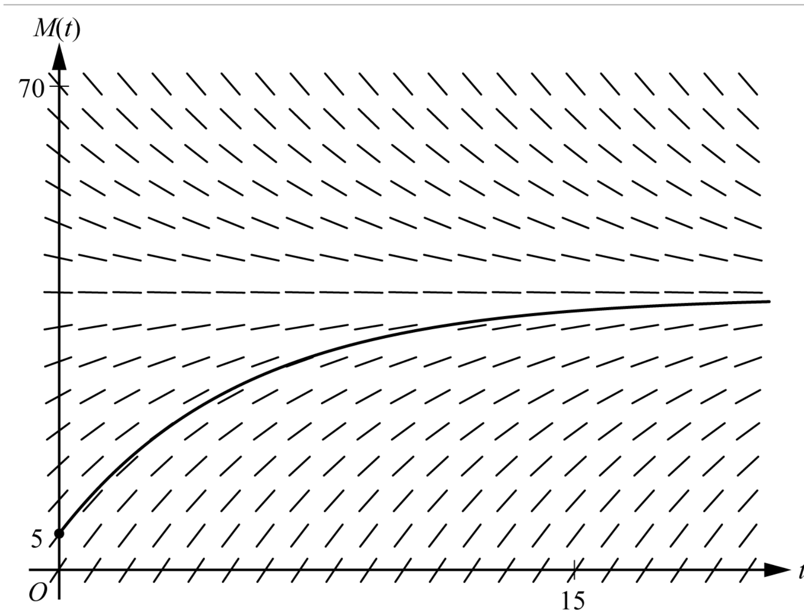
Part B (AB or BC): Graphing calculator not allowed
Question 3 — 9 points

General Scoring Notes

The model solution is presented using standard mathematical notation.

Answers (numeric or algebraic) need not be simplified. Answers given as a decimal approximation should be correct to three places after the decimal point. Within each individual free-response question, at most one point is not earned for inappropriate rounding.

A bottle of milk is taken out of a refrigerator and placed in a pan of hot water to be warmed. The increasing function M models the temperature of the milk at time t, where $M(t)$ is measured in degrees Celsius (°C) and t is the number of minutes since the bottle was placed in the pan. M satisfies the differential equation $\frac{dM}{dt} = \frac{1}{4}(40 - M)$. At time $t = 0$, the temperature of the milk is 5°C. It can be shown that $M(t) < 40$ for all values of t.

	Model Solution	Scoring	
(a)	A slope field for the differential equation $\frac{dM}{dt} = \frac{1}{4}(40 - M)$ is shown. Sketch the solution curve through the point $(0, 5)$.		
		Solution curve	**1 point**

Scoring notes:

- The solution curve must pass through the point $(0, 5)$, extend reasonably close to the left and right edges of the rectangle, and have no obvious conflicts with the given slope lines.
- Only portions of the solution curve within the given slope field are considered.
- The solution curve must lie entirely below the horizontal line segments at $M = 40$.

Total for part (a) — 1 point

(b) Use the line tangent to the graph of M at $t = 0$ to approximate $M(2)$, the temperature of the milk at time $t = 2$ minutes.

$\left.\frac{dM}{dt}\right\|_{t=0} = \frac{1}{4}(40 - 5) = \frac{35}{4}$	$\left.\frac{dM}{dt}\right\|_{t=0}$	**1 point**
The tangent line equation is $y = 5 + \frac{35}{4}(t - 0)$. $M(2) \approx 5 + \frac{35}{4} \cdot 2 = 22.5$ The temperature of the milk at time $t = 2$ minutes is approximately 22.5° Celsius.	Approximation	**1 point**

Scoring notes:

- The value of the slope may appear in a tangent line equation or approximation.
- A response of $5 + \frac{35}{4} \cdot 2$ is the minimal response to earn both points.
- A response of $\frac{1}{4}(40 - 5)$ earns the first point. If there are any subsequent errors in simplification, the response does not earn the second point.
- In order to earn the second point the response must present an approximation found by using a tangent line that:
  - passes through the point $(0, 5)$ and
  - has slope $\frac{35}{4}$ or a nonzero slope that is declared to be the value of $\frac{dM}{dt}$.
- An unsupported approximation does not earn the second point.
- The approximation need not be simplified, but the response does not earn the second point if the approximation is simplified incorrectly.

Total for part (b) 2 points

(c) Write an expression for $\frac{d^2M}{dt^2}$ in terms of M. Use $\frac{d^2M}{dt^2}$ to determine whether the approximation from part (b) is an underestimate or an overestimate for the actual value of $M(2)$. Give a reason for your answer.

$\frac{d^2M}{dt^2} = -\frac{1}{4}\frac{dM}{dt} = -\frac{1}{4}\left(\frac{1}{4}(40 - M)\right) = -\frac{1}{16}(40 - M)$	$\frac{d^2M}{dt^2}$	**1 point**
Because $M(t) < 40$, $\frac{d^2M}{dt^2} < 0$, so the graph of M is concave down. Therefore, the tangent line approximation of $M(2)$ is an overestimate.	Overestimate with reason	**1 point**

Scoring notes:

- The first point is earned for either $\frac{d^2M}{dt^2} = -\frac{1}{4}\left(\frac{1}{4}(40 - M)\right)$ or $\frac{d^2M}{dt^2} = -\frac{1}{16}(40 - M)$ (or equivalent). A response that presents any subsequent simplification error does not earn the second point.
- A response that presents an expression for $\frac{d^2M}{dt^2}$ in terms of $\frac{dM}{dt}$ but fails to continue to an expression in terms of M (i.e., $\frac{d^2M}{dt^2} = -\frac{1}{4}\frac{dM}{dt}$) does not earn the first point but is eligible for the second point.
- If the response presents an expression for $\frac{d^2M}{dt^2}$ that is incorrect, the response is eligible for the second point only if the expression is a nonconstant linear function that is negative for $5 < M < 40$.
  - Special case: A response that presents $\frac{d^2M}{dt^2} = \frac{1}{16}(40 - M)$ does not earn the first point but is eligible to earn the second point for a consistent answer and reason.
- To earn the second point a response must include $\frac{d^2M}{dt^2} < 0$, or $\frac{dM}{dt}$ is decreasing, or the graph of M is concave down, as well as the conclusion that the approximation is an overestimate.
- A response that presents an argument based on $\frac{d^2M}{dt^2}$ or concavity at a single point does not earn the second point.

Total for part (c) 2 points

(d) Use separation of variables to find an expression for $M(t)$, the particular solution to the differential equation $\frac{dM}{dt} = \frac{1}{4}(40 - M)$ with initial condition $M(0) = 5$.

$\frac{dM}{40 - M} = \frac{1}{4}dt$ $\int \frac{dM}{40 - M} = \int \frac{1}{4}\,dt$	Separates variables	**1 point**
$-\ln\lvert 40 - M\rvert = \frac{1}{4}t + C$	Finds antiderivatives	**1 point**
$-\ln\lvert 40 - 5\rvert = 0 + C \Rightarrow C = -\ln 35$ $M(t) < 40 \Rightarrow 40 - M > 0 \Rightarrow \lvert 40 - M\rvert = 40 - M$ $-\ln(40 - M) = \frac{1}{4}t - \ln 35$ $\ln(40 - M) = -\frac{1}{4}t + \ln 35$	Constant of integration and uses initial condition	**1 point**

$40 - M = 35e^{-t/4}$ $M = 40 - 35e^{-t/4}$	Solves for M	**1 point**

Scoring notes:

- A response with no separation of variables earns 0 out of 4 points.
- A response that presents an antiderivative of $-\ln(40 - M)$ without absolute value symbols is eligible for all 4 points.
- A response with no constant of integration can earn at most the first 2 points.
- A response is eligible for the third point only if it has earned the first 2 points.
  - Special Case: A response that presents $+\ln(40 - M) = \frac{t}{4} + C$ (or equivalent) does not earn the second point, is eligible for the third point, but not eligible for the fourth.
- An eligible response earns the third point by correctly including the constant of integration in an equation and substituting 0 for t and 5 for M.
- A response is eligible for the fourth point only if it has earned the first 3 points.
- A response earns the fourth point only for an answer of $M = 40 - 35e^{-t/4}$ or equivalent.

Total for part (d) 4 points

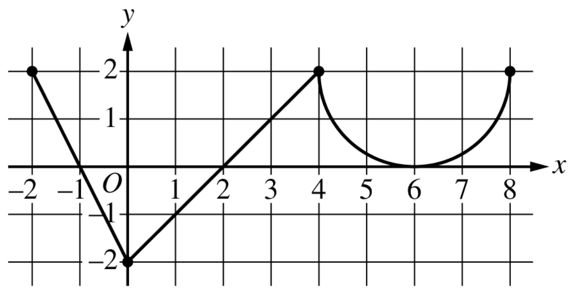
Total for question 3 9 points

Part B (AB or BC): Graphing calculator not allowed
Question 4 — 9 points

General Scoring Notes

The model solution is presented using standard mathematical notation.

Answers (numeric or algebraic) need not be simplified. Answers given as a decimal approximation should be correct to three places after the decimal point. Within each individual free-response question, at most one point is not earned for inappropriate rounding.



Graph of f'

The function f is defined on the closed interval $[-2, 8]$ and satisfies $f(2) = 1$. The graph of f', the derivative of f, consists of two line segments and a semicircle, as shown in the figure.

	Model Solution	Scoring	
(a)	Does f have a relative minimum, a relative maximum, or neither at $x = 6$? Give a reason for your answer.		
	$f'(x) > 0$ on $(2, 6)$ and $f'(x) > 0$ on $(6, 8)$. $f'(x)$ does not change sign at $x = 6$, so there is neither a relative maximum nor a relative minimum at this location.	Answer with reason	**1 point**

Scoring notes:

- A response that declares $f'(x)$ does not change sign at $x = 6$, so neither, is sufficient to earn the point.
- A response does not have to present intervals on which $f'(x)$ is positive or negative, but if any are given, they must be correct. Any presented intervals may include none, one, or both endpoints.
- A response that declares $f'(x) > 0$ before and after $x = 6$ does not earn the point.

Total for part (a) — 1 point

(b) On what open intervals, if any, is the graph of f concave down? Give a reason for your answer.

The graph of f is concave down on $(-2, 0)$ and $(4, 6)$ because f' is decreasing on these intervals.	Intervals	**1 point**
	Reason	**1 point**

Scoring notes:

- The first point is earned only by an answer of $(-2, 0)$ and $(4, 6)$, or these intervals including one or both endpoints.
- A response must earn the first point to be eligible for second point.
- To earn the second point a response must correctly discuss the behavior of f' or the slopes of f'.
- Special case: A response that presents exactly one of the two correct intervals with a correct reason earns 1 out of 2 points.

Total for part (b) **2 points**

(c) Find the value of $\lim_{x \to 2} \frac{6f(x) - 3x}{x^2 - 5x + 6}$, or show that it does not exist. Justify your answer.

Because f is differentiable at $x = 2$, f is continuous at $x = 2$, so $\lim_{x \to 2} f(x) = f(2) = 1$. $\lim_{x \to 2} (6f(x) - 3x) = 6 \cdot 1 - 3 \cdot 2 = 0$ $\lim_{x \to 2} \left(x^2 - 5x + 6\right) = 0$	Limits of numerator and denominator	**1 point**
Because $\lim_{x \to 2} \frac{6f(x) - 3x}{x^2 - 5x + 6}$ is of indeterminate form $\frac{0}{0}$, L'Hospital's Rule can be applied. Using L'Hospital's Rule, $\lim_{x \to 2} \frac{6f(x) - 3x}{x^2 - 5x + 6} = \lim_{x \to 2} \frac{6f'(x) - 3}{2x - 5} = \frac{6 \cdot 0 - 3}{2 \cdot 2 - 5} = 3.$	Uses L'Hospital's Rule	**1 point**
	Answer	**1 point**

Scoring notes:

- The first point is earned by the presentation of two separate limits for the numerator and denominator.
- A response that presents a limit explicitly equal to $\frac{0}{0}$ does not earn the first point.
- The second point is earned by applying L'Hospital's Rule, that is, by presenting at least one correct derivative in the limit of a ratio of derivatives.
- The third point is earned for the correct answer with supporting work.

Total for part (c) **3 points**

(d) Find the absolute minimum value of f on the closed interval $[-2, 8]$. Justify your answer.

$f'(x) = 0 \Rightarrow x = -1,\ x = 2,\ x = 6$	Considers $f'(x) = 0$	**1 point**
The function f is continuous on $[-2, 8]$, so the candidates for the location of an absolute minimum for f are $x = -2,\ x = -1,\ x = 2,\ x = 6,$ and $x = 8$.	Justification	**1 point**
	Answer	**1 point**

x	$f(x)$
-2	3
-1	4
2	1
6	$7 - \pi$
8	$11 - 2\pi$

The absolute minimum value of f is $f(2) = 1$.

Scoring notes:

- To earn the first point a response must state $f' = 0$ or equivalent. Listing the zeros of f' is not sufficient.
- A response that presents any error in evaluating f at any critical point or endpoint will not earn the justification point.
- A response need not present the value of $f(-1)$ provided $x = -1$ is eliminated because it is the location of a local maximum. A response need not present the value of $f(6)$ provided $x = 6$ is eliminated by reference to part (a) or eliminated through analysis.
- A response need not present the value of $f(8)$ provided there is a presentation that argues $f'(x) \geq 0$ for $x > 2$ and, therefore, $f(8) > f(2)$.
- A response that does not consider both endpoints does not earn the justification point.
- The answer point is earned only for indicating that the minimum value is 1. It is not earned for noting that the minimum occurs at $x = 2$.

Total for part (d) **3 points**

Total for question 4 **9 points**

Part B (AB): Graphing calculator not allowed
Question 5 — 9 points

General Scoring Notes

The model solution is presented using standard mathematical notation.

Answers (numeric or algebraic) need not be simplified. Answers given as a decimal approximation should be correct to three places after the decimal point. Within each individual free-response question, at most one point is not earned for inappropriate rounding.

x	0	2	4	7
$f(x)$	10	7	4	5
$f'(x)$	$\frac{3}{2}$	−8	3	6
$g(x)$	1	2	−3	0
$g'(x)$	5	4	2	8

The functions f and g are twice differentiable. The table shown gives values of the functions and their first derivatives at selected values of x.

Model Solution	Scoring	

(a) Let h be the function defined by $h(x) = f(g(x))$. Find $h'(7)$. Show the work that leads to your answer.

Model Solution	Scoring	
$h'(x) = f'(g(x)) \cdot g'(x)$ $h'(7) = f'(g(7)) \cdot g'(7)$	Chain rule	**1 point**
$= f'(0) \cdot 8 = \frac{3}{2} \cdot 8 = 12$	Answer	**1 point**

Scoring notes:

- The first point is earned for either $h'(x) = f'(g(x)) \cdot g'(x)$ or $h'(7) = f'(g(7)) \cdot g'(7)$.
- If the first point is earned, the second point is earned only for an answer of 12 (or equivalent).
- If the first point is not earned, the second point can be earned only for a response of either $f'(0) \cdot 8 = 12$ or $\frac{3}{2} \cdot 8$.
- A response of 12 with no supporting work does not earn either point.

Total for part (a) — 2 points

(b) Let k be a differentiable function such that $k'(x) = (f(x))^2 \cdot g(x)$. Is the graph of k concave up or concave down at the point where $x = 4$? Give a reason for your answer.

Model Solution	Scoring	
$k''(x) = 2f(x) \cdot f'(x) \cdot g(x) + (f(x))^2 \cdot g'(x)$	Product or chain rule	**1 point**

$k''(4) = 2f(4) \cdot f'(4) \cdot g(4) + (f(4))^2 \cdot g'(4)$	
$= 2 \cdot 4 \cdot 3 \cdot (-3) + 4^2 \cdot 2 = -72 + 32 = -40$	$k''(4)$ **1 point**
The graph of k is concave down at the point where $x = 4$ because $k''(4) < 0$ and k'' is continuous.	Answer with reason **1 point**

Scoring notes:

- The first point is earned for either $k''(x) = 2f(x) \cdot f'(x) \cdot g(x) + (f(x))^2 \cdot g'(x)$ or $k''(4) = 2f(4) \cdot f'(4) \cdot g(4) + (f(4))^2 \cdot g'(4)$.
- The first point is also earned by any of the following incorrect expressions, each of which has a single error in the application of the product rule or the chain rule:
  - $2f(x) \cdot g(x) + (f(x))^2 \cdot g'(x)$ or $2f(4) \cdot g(4) + (f(4))^2 \cdot g'(4)$
  - $2f'(x) \cdot g(x) + (f(x))^2 \cdot g'(x)$ or $2f'(4) \cdot g(4) + (f(4))^2 \cdot g'(4)$
  - $f'(x) \cdot g(x) + (f(x))^2 \cdot g'(x)$ or $f'(4) \cdot g(4) + (f(4))^2 \cdot g'(4)$
  - $2f(x) \cdot f'(x) \cdot g'(x)$ or $2f(4) \cdot f'(4) \cdot g'(4)$
  - Note: A response that presents one of these expressions cannot earn the second point.
- To earn the second point a response must correctly find $k''(4) = -40$ (or equivalent) with supporting work.
- The third point is earned for an answer and reason that are consistent with any declared nonzero value of $k''(4)$.

Total for part (b) 3 points

(c) Let m be the function defined by $m(x) = 5x^3 + \int_0^x f'(t)\,dt$. Find $m(2)$. Show the work that leads to your answer.

$m(2) = 5 \cdot 8 + \int_0^2 f'(t)\,dt = 40 + (f(2) - f(0))$ $= 40 + (7 - 10) = 37$	Answer with supporting work **1 point**

Scoring notes:

- The point is earned only for an answer of 37 (or equivalent) with supporting work equivalent to $5 \cdot 8 + (f(2) - f(0))$, $40 + (f(2) - f(0))$, $5 \cdot 8 + (7 - 10)$, or $40 + (7 - 10)$.
- An answer of 37 with no supporting work does not earn the point.

Total for part (c) 1 point

(d) Is the function m defined in part (c) increasing, decreasing, or neither at $x = 2$? Justify your answer.

$m'(x) = 15x^2 + f'(x)$	Considers $m'(x)$ **1 point**
$m'(2) = 15 \cdot 4 + f'(2) = 60 + (-8) = 52$	$m'(2)$ with supporting work **1 point**

The graph of m is increasing at $x = 2$ because $m'(2) > 0$.	Answer with justification	**1 point**

Scoring notes:

- The first point is earned for considering $m'(x)$, $m'(2)$, or m'. This consideration may appear in a justification statement.
- The second point is earned for $m'(2) = 15 \cdot 2^2 + f'(2)$, $m'(2) = 60 + f'(2)$, or $m'(2) = 60 - 8$ but is not earned for an unsupported response of $m'(2) = 52$.
- The third point is earned for an answer and justification consistent with any declared value of $m'(2)$.

Total for part (d) 3 points

Total for question 5 9 points

Part B (AB): Graphing calculator not allowed
Question 6 — 9 points

General Scoring Notes
The model solution is presented using standard mathematical notation.

Answers (numeric or algebraic) need not be simplified. Answers given as a decimal approximation should be correct to three places after the decimal point. Within each individual free-response question, at most one point is not earned for inappropriate rounding.

Consider the curve given by the equation $6xy = 2 + y^3$.

	Model Solution	Scoring	
(a)	Show that $\frac{dy}{dx} = \frac{2y}{y^2 - 2x}$.		
	$\frac{d}{dx}(6xy) = \frac{d}{dx}\left(2 + y^3\right) \Rightarrow 6y + 6x\frac{dy}{dx} = 3y^2\frac{dy}{dx}$	Implicit differentiation	**1 point**
	$\Rightarrow 2y = \frac{dy}{dx}\left(y^2 - 2x\right) \Rightarrow \frac{dy}{dx} = \frac{2y}{y^2 - 2x}$	Verification	**1 point**

Scoring notes:

- The first point is earned only for the correct implicit differentiation of $6xy = 2 + y^3$. Responses may use alternative notations for $\frac{dy}{dx}$, such as y'.
- The second point cannot be earned without the first point.
- It is sufficient to present $2y = \frac{dy}{dx}\left(y^2 - 2x\right)$ to earn the second point, provided there are no subsequent errors.

Total for part (a) 2 points

	Model Solution	Scoring	
(b)	Find the coordinates of a point on the curve at which the line tangent to the curve is horizontal, or explain why no such point exists.		
	For the line tangent to the curve to be horizontal, it is necessary that $2y = 0$ (so $y = 0$) and that $y^2 - 2x \neq 0$.	Sets $2y = 0$	**1 point**
	Substituting $y = 0$ into $6xy = 2 + y^3$ yields the equation $6x \cdot 0 = 2$, which has no solution. Therefore, there is no point on the curve at which the line tangent to the curve is horizontal.	Answer with reason	**1 point**

Scoring notes:

- The first point is earned with any of $2y = 0$, $y = 0$, $\frac{dy}{dx} = 0$, $dy = 0$, $y' = 0$, or $\frac{2y}{y^2 - 2x} = 0$.
- A response need not state that at a horizontal tangent, $y^2 - 2x \neq 0$.

Total for part (b) 2 points

(c) Find the coordinates of a point on the curve at which the line tangent to the curve is vertical, or explain why no such point exists.

For a line tangent to this curve to be vertical, it is necessary that $2y \neq 0$ and that $y^2 - 2x = 0$ (so $x = \frac{y^2}{2}$).	Sets $y^2 - 2x = 0$	**1 point**
Substituting $x = \frac{y^2}{2}$ into $6xy = 2 + y^3$ yields the equation $3y^2 \cdot y = 2 + y^3 \Rightarrow 2y^3 = 2 \Rightarrow y = 1$.	Substitutes $x = \frac{y^2}{2}$ into $6xy = 2 + y^3$	**1 point**
Substituting $y = 1$ in $6xy = 2 + y^3$ yields $6x = 2 + 1$, or $x = \frac{1}{2}$. The tangent line to the curve is vertical at the point $\left(\frac{1}{2}, 1\right)$.	Answer	**1 point**

Scoring notes:

- The first point can be earned by presenting $y^2 = 2x$ or $y = \sqrt{2x}$.
- The second point can be earned for the substitution of $y = \sqrt{2x}$ into $6xy = 2 + y^3$, or for substituting $x = \frac{2 + y^3}{6y}$ into $y^2 - 2x = 0$.
- A response earns all three points by setting $y^2 - 2x = 0$, declaring the point $\left(\frac{1}{2}, 1\right)$, and verifying that this point is on the curve $6xy = 2 + y^3$.
- A response that identifies the point $\left(\frac{1}{2}, 1\right)$ but does not verify that the point is on the curve, does not earn the second or the third point.
- To earn the third point the response must present both coordinates of the point $\left(\frac{1}{2}, 1\right)$. The coordinates need not appear as an ordered pair as long as they are labeled.

Total for part (c) 3 points

(d) A particle is moving along the curve. At the instant when the particle is at the point $\left(\frac{1}{2}, -2\right)$, its horizontal position is increasing at a rate of $\frac{dx}{dt} = \frac{2}{3}$ unit per second. What is the value of $\frac{dy}{dt}$, the rate of change of the particle's vertical position, at that instant?

$6y\frac{dx}{dt} + 6x\frac{dy}{dt} = 0 + 3y^2\frac{dy}{dt}$	Uses implicit differentiation with respect to t	**1 point**
At the point $(x, y) = \left(\frac{1}{2}, -2\right)$, $6(-2)\left(\frac{2}{3}\right) + 6\left(\frac{1}{2}\right)\frac{dy}{dt} = 3(-2)^2\frac{dy}{dt}$ $\Rightarrow -8 + 3\frac{dy}{dt} = 12\frac{dy}{dt}$ $\Rightarrow \frac{dy}{dt} = -\frac{8}{9}$ unit per second	Answer	**1 point**

Scoring notes:

- The first point is earned by presenting one or more of the terms $6y\frac{dx}{dt}$, $6x\frac{dy}{dt}$, or $3y^2\frac{dy}{dt}$.
- Units will not affect scoring in this part.
- An unsupported response of $-\frac{8}{9}$ earns no points.
- Alternate solution:

$$\frac{dy}{dt} = \frac{dy}{dx} \cdot \frac{dx}{dt}$$

$$\left.\frac{dy}{dx}\right|_{(x,\, y)=(1/2,\, -2)} = \frac{2(-2)}{(-2)^2 - 2(1/2)} = -\frac{4}{3}$$

$$\left.\frac{dy}{dt}\right|_{(x,\, y)=(1/2,\, -2)} = \left.\frac{dy}{dx} \cdot \frac{dx}{dt}\right|_{(x,\, y)=(1/2,\, -2)} = -\frac{4}{3} \cdot \frac{2}{3} = -\frac{8}{9} \text{ unit per second}$$

  - The first point is earned for the statement $\frac{dy}{dt} = \frac{dy}{dx} \cdot \frac{dx}{dt}$ or equivalent.
  - A numerical expression, such as $-\frac{4}{3} \cdot \frac{2}{3}$ or $\frac{2(-2)}{(-2)^2 - 2\left(\frac{1}{2}\right)} \cdot \frac{2}{3}$, earns both points.

Total for part (d) 2 points

Total for question 6 9 points